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STRUCTURE OF HIGH MACH-NUMBER SHOCKS IN LOW-BETA PLASMAS

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ABSTRACT

High Mach number shock waves in low-beta plasmas are discussed. It is suggested that most of the ion heating in such shocks occurs through a thin electrostatic shock transition embedded within the broader magnetic shock structure. This electrostatic shock would consist of a layer of high intensity ion sound turbulence with propagation vectors concentrated in the ion flow direction. It would be distinct from turbulence propagating in the transverse direction associated with anomalous electrical resistance.

In previous investigations[1] of collisionless shock waves in high-beta plasmas it was pointed out that under some circumstances such shock transitions are expected to consist of a thin ion-sound electrostatic shock together with an attached much broader magnetic structure. Recent numerical integration of the two-fluid equations (including ion viscosity) by Robson and Sheffield[2] and by Macmahon[3] sometimes appears to produce a similar separation of the shock into a thin layer of viscous ion heating and a broader magnetic structure. In this note we argue that a general phenomenon is involved and that at high Mach-numbers in low-beta plasmas the principal ion heating occurs through an electrostatic shock embedded within the broader laminar magnetic structure. Such a situation, should it be supported by experimental observations, would result in a considerable simplification in our theoretical view of such high speed shocks.

It is well known that above a critical Mach number (∿2 or 3) two-fluid models for magnetosonic shocks, which include electrical resistance as the only dissipative mechanism, undergo a wave-breaking phenomenon[4]. This derives from the electrostatic field E_x (see Fig.1) which can become large enough to reflect the ion stream forward. From a more realistic description based on the Vlasov-equation[5] it appears that on approach (from below) to this critical Mach number a fraction of ions are reflected forward and produce a "foot" structure in front of the magnetosonic shock.

If however a viscosity term is introduced into the fluid models, for example by adding a term $- \mu \, d^2V_x/dx^2$ to the right side of the ion x-momentum equation (and adding an equation of state to account for viscous ion heating), then breaking does not occur and high Mach-number solutions can be obtained[2,3]. This results mathematically from the steep ion velocity gradients with attendant ion heating just prior to wave breaking.

The above device however appears somewhat artificial. It raises the obvious questions of the physical origin of the viscosity in the absence of classical collisions, whether it is correct to concentrate viscous effects near the wave breaking point, or whether the concept of anomalous viscosity embodied in a term $-\mu\, d^2V_x/dx^2$ is relevant at all.

If electrostatic micro-instabilities are responsible for anomalous ion viscous effects (and presumably other equally important transport effects resulting from ion heating such as spatial diffusion, etc.) then it is likely that the waves involved have $\underline{k}$ vectors concentrated in the x-direction. They are to be distinguished from waves causing anomalous resistivity which propagate mainly in the y direction and derive from the diamagnetic electron current.

One could envisage at least two situations. First it is possible that "viscous" electrostatic turbulence is smeared over the entire magnetic shock thickness giving a steady ion heating throughout the shock.

A second possibility is that the electrostatic turbulence which principally heats the ions occurs in a thin high-intensity layer with length scales of several times $L_D = v_e/\omega_e << c/\omega_e$, i.e., the equivalent of an electrostatic shock embedded within the magnetic shock ($v_e = \sqrt{KT_e/m}$). We shall give arguments in support of this second assumption and discuss the location of such an electrostatic shock. These arguments are by necessity non-rigorous and an experimental determination of the spatial distribution of turbulence in the flow direction should be made.

We start by considering a laminar Vlasov-magnetosonic[5] shock which is on the brink of the fluid breaking Mach number. The situation is shown in Fig. 1 and we see that a substantial number of ions will be reflected. The ion distributions close to the first maximum $(x = x_M - \delta)$ and at x_1 are shown.

Now if $T_e \gg T_i$ at x_1, the distribution at x_1 drives unstable ion waves with growth rates $\sim \omega_i = (4\pi Ne^2/M)^{1/2}$. Provided $B^2/4\pi Nmc^2 \ll 1$ we have $\omega_i \gg \sqrt{\Omega_e \Omega_i}$ so that the instability would grow rapidly[6] $(\Omega_e = eB/mc)$. We assume that in a small fraction of a transit time through the shock the distribution at x_1 converts through instability into a hot ion gas as shown in 1c. (Reflected ions to the left of x_1 are now imagined to be removed since we are attempting to infer the final steady state after instability and these ions must continually originate from the ion population at x_1).

Now some of the hot ion gas at x_1 will drift slowly down stream and some attempt to expand upstream. (Inside the magnetosonic wave only electrostatic forces influence the ions and this tends to accelerate ions traveling upstream.) However in expanding upstream the hot ion gas will encounter the incoming cold ion stream which will produce an unstable situation (left of Fig.1c) in the front edge of the hot layer. Incoming ions will continually plow into the hot ion gas and regenerate it, and this situation will maintain itself as an electrostatic shock transition[1] as in Fig. 1d. If finally we imagine that the Mach number is slowly turned up it appears that this situation is more easily maintained at very high Mach numbers.

It is interesting to note that for Mach numbers well above the critical Mach number, a magnetosonic shock transforms a low-β plasma ($\beta << 1$) into a high-β plasma ($\beta \gtrsim 1$). The Rankine-Hugoniot relations give

$$\beta_2 = \frac{\Sigma\, N_2\, KT_2}{B_2^2/8\pi} = \frac{3}{32}\, M_A^2 \qquad . \tag{1}$$

For example $\beta_2 \sim 2.5$ for $M_A = 5$. A large fraction of this transition may occur at the electrostatic shock. The beta of the downstream plasma for an oblique shock is even higher.

Next consider the question of where such an electrostatic shock might be located within the magnetic structure. Since we have assumed that ion sound waves (appropriately modified for the effects of $\underline{B}$ on the electron dynamics) are responsible for the transition, we require a condition $T_e/T_i = \alpha >$ unity to be reached. The plasma must therefore be suitably "primed" by the forward portion of the shock. This "priming" consists of processes such as anomalous resistance which increase T_e/T_i to the critical value α (eg. $\alpha \sim 5$) in the front part of the magnetic shock. We assume that the hot high-β downstream ions will diffuse as far forward as possible consistent with their local regeneration for a steady state. Equivalently we assume that the electrostatic transition occurs as soon as $T_e/T_i = \alpha$.

If such electrostatic transitions occur the magnetic field variation through a high Mach-number shock would appear <u>laminar</u> and the calculational procedure would typically be as follows: First integrate the two-fluid equations with anomalous resistance through the shock until $T_e/T_i = \alpha$. Then make an electrostatic transition using the Rankine-Hugoniot relations. Then continue integrating through the shock using two-fluid equations for a hot plasma.

As an example of how this would work consider a magnetosonic shock. Throughout such a transition the following fluxes are constant,

$$NV_x = \text{const.} \quad , \tag{2}$$

$$MN\left(\bar{v}^2 + V_x^2\right) + \frac{1}{8\pi}\left(B^2 - E_x^2\right) = \text{x-momentum flux,} \tag{3}$$

$$NV_x\left(5\,\bar{v}^2 + V_x^2\right) + \frac{V_x\,B^2}{2\pi M} = \text{x-energy flux,} \tag{4}$$

$$V_x\left(mNV_{ye} + MNV_{yi}\right) - \frac{1}{4\pi}E_y\,E_x = \text{y-momentum flux,} \tag{5}$$

where local Maxwellians have been assumed with equal ion and electron densities N, and

$$\bar{v}^2 = v_i^2 + \frac{m}{M}v_e^2$$

with $v_{i,e}^2 = KT_{i,e}/(M,m)$, B the z-magnetic field, E_x the electrostatic field, and $\underline{V}$ the flow velocities. These quantities are all functions of x except for $E_y = V_x(-\infty)\,B(-\infty)/c$ = constant in the shock frame (Fig.2a). Equations (3) - (5) omit turbulent pressures $\langle \delta \underline{E}^2 \rangle$

which could become large only inside the electrostatic transition.

The electrostatic field in the magnetosonic wave is

$$E_x \cong \frac{1}{eN} \frac{\partial P_e}{\partial x} - \frac{B}{c} V_{ye} \tag{7}$$

where $P_{i,e} = NKT_{i,e}$. From (5) it also follows that $V_{ye} \cong - MV_{yi}/m$ provided $B^2/4\pi Nmc^2 << 1$.

Now consider the jump in B across the electrostatic shock. It is of order $\Delta B \sim - (4\pi e/c)\, NV_{ye}\Delta x$ where Δx is the shock thickness. Assuming the electron drift velocity V_{ye} is limited to not more than $\sim v_e$ by instabilities we see that $\Delta B/B \sim \Delta x/(c/\omega_e) << 1$. We thus assume $B_{1s} = B_{2s}$ across the inner shock.

Since the laminar electrostatic pressures in (3) and (4) are small, and the pressure differential in $B^2/8\pi$ is small, the jump conditions for the electrostatic shock are approximately obtained by dropping the field terms in (2) - (4). The resulting equations yield

$$\frac{N_{2s}}{N_{1s}} = \frac{V_{1s}}{V_{2s}} = \frac{4}{\left(1 + \frac{5\bar{v}_{1s}^2}{V_{1s}^2}\right)} \tag{8}$$

$$\frac{\bar{v}_{2s}^2}{V_{1s}^2} = \frac{1}{16} \left(3 - \frac{\bar{v}_{1s}^2}{V_{1s}^2}\right)\left(1 + \frac{5\bar{v}_{1s}^2}{V_{1s}^2}\right) \quad , \tag{9}$$

where subscripts 1s and 2s have been used to denote parameters evaluated just upstream and downstream from the electrostatic shock (Fig. 2a).

If next we assume there is no total charge on the shock it follows that $E_{x1s} = E_{x2s}$ (although E_x, as well as $\delta\underline{E}$, probably becomes large inside the transition). Thus since from (7), $E_x \cong - BV_{ye}/c$, it follows from Maxwell's equation $\partial B/\partial x = 4\pi NeV_{ye}/c$ that

$$\left(\frac{\partial B}{\partial x}\right)_{2s} \cong \left(\frac{\partial B}{\partial x}\right)_{1s} \left(\frac{N_{2s}}{N_{1s}}\right) \quad . \tag{10}$$

An increase in the slope of B is therefore expected to occur just behind the electrostatic transition as shown in Fig. 2a.

We also note at this point that some hot ions may leak through the turbulent electrostatic shock (they are not reflected from the front) and produce a foot structure as shown in Fig. 2a even at high Mach numbers. This phenomenon was discussed in reference (1).

For oblique whistler shocks (Fig. 2b) the "priming region" would consist of several oscillations before T_e/T_i increased to the critical value α, because the whistler waves in this case damp out upstream.

In conclusion we emphasize that it is important to experimentally determine the spatial extent and location of electrostatic turbulence with $\underline{k}$ vectors along x in high Mach-number shocks. If the arguments in this paper are correct then the location of the electrostatic transition within the magnetic shock is expected to be sensitive to the ratio T_e/T_i in the upstream ambient plasma. The plasma parameters should also be chosen so that there is a clear separation between the length scales involved. Assuming the electrostatic shock has a thickness[(1)] L_s = few x $(3/5)^{1/2}$ $[V_{1s}/(KT_{ei}/M)^{1/2}]$ L_D then it is clear that the separation of scales is much better for oblique shock. $(L_s << c/\omega_i)$ than for magnetosonic shocks $(L_s < c/\omega_e)$. In the latter case the magnetic and electrostatic length scales may sometimes tend to blur together.

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REFERENCES.

1. D.A. Tidman, Phys.Fluids 10, 547 (1967). The model in this paper needs to be modified by inclusion of ballistic effects on the spatial evolution of the turbulence. See "Comments on Turbulent Shocks" by D. Biskamp and D. Pfirsch, and "Reply to Comments" by D.A. Tidman and N.A. Krall, Phys.Fluids (in press) 1968. See also discussion in "Ballistic Wake of Turbulence in a Plasma Shock Wave", N.A. Krall and D.A. Tidman, Phys.Fluids (in press) 1968.
2. A.E. Robson and J. Sheffield, Oblique Hydromagnetic Shock Waves, Univ. of Texas, Tech. Note CN-24/A-6 (1968).
3. A.B. Macmahon, abstract 3c-4, Bull.A.Phys.Soc. 13, No. 11 (1968).
4. See for example R.Z. Sagdeev, in Reviews of Plasma Physics, Vol. IV, ed. by M.A. Leontovich, translated from the Russian by Consultants Bureau, New York, 1966.
5. This can be obtained by a simple modification of: C.S. Morawetz, Phys. Fluids 4, 988 (1961), Phys.Fluids 5, 1447 (1962), or N.A. Krall, Phys.Fluids (in press), 1968. See also H. Vernon Wong, abstract 3c-2, Bull.Am.Phys.Soc. 13, No 11 (1968).
6. Considering $f_i = \frac{N}{2}\,[\delta(v - V_x)] + \delta(v + V_x)]$ and Maxwellian electrons it turns out that the wave-number range $0 < k^2 < \omega_i^2(V_x^{-2} - c_s^{-2})$ is unstable, where $c_s = (KT_e/M)^{\frac{1}{2}}$. The instability therefore only exists if $V_x < c_s$. The e-folding length available, before the electrostatic field E_x accelerates reflected ions to a velocity in excess of c_s, is $\ell \sim c_s/(dV_x/dx) \sim (c/\omega_e)\sqrt{\beta_e}/M_A$. We have assumed $\ell >> L_D$. It should be added however that the unstable ion wave has only a small group or phase velocity in the shock frame which should reduce the stringency of this condition.

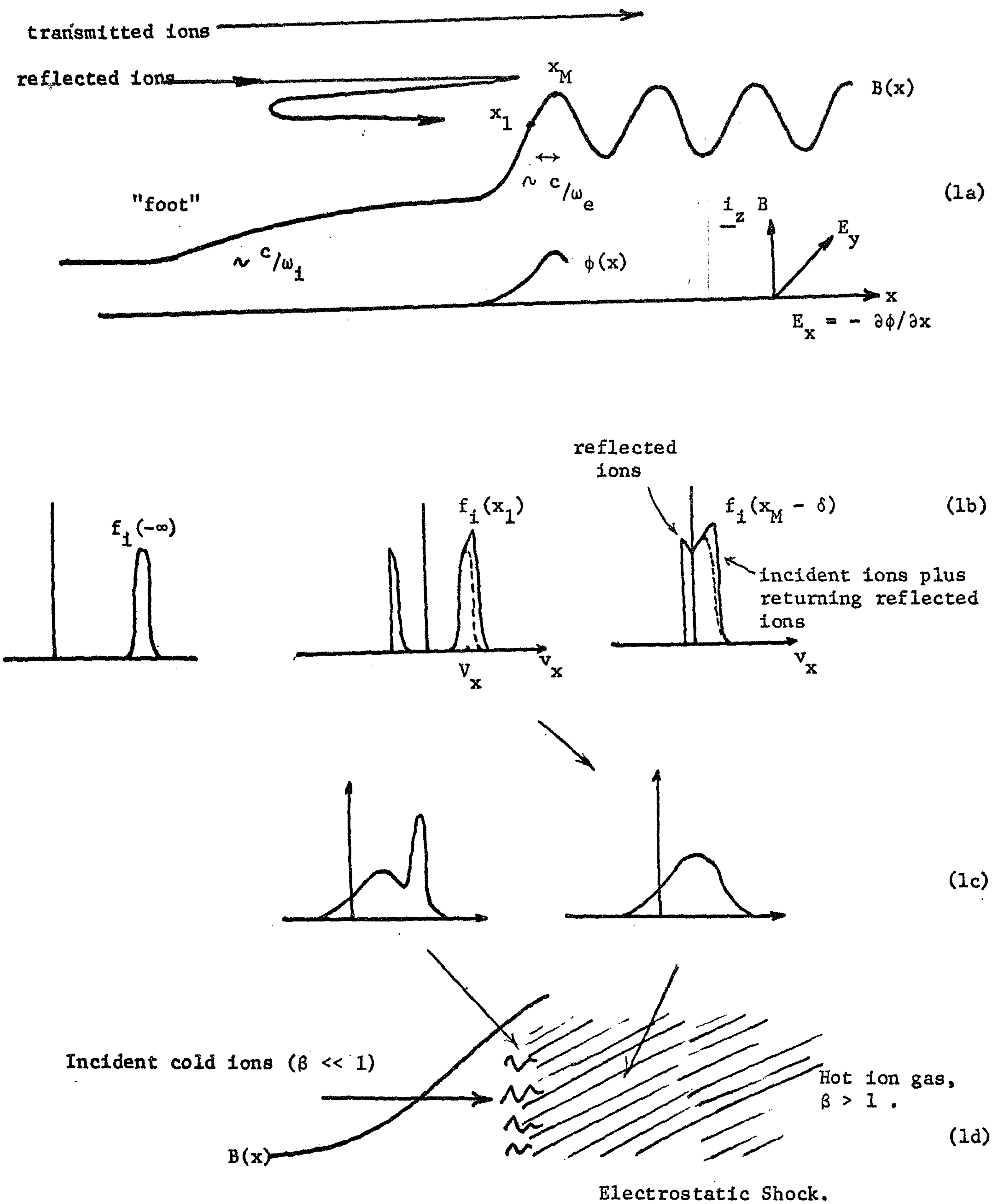
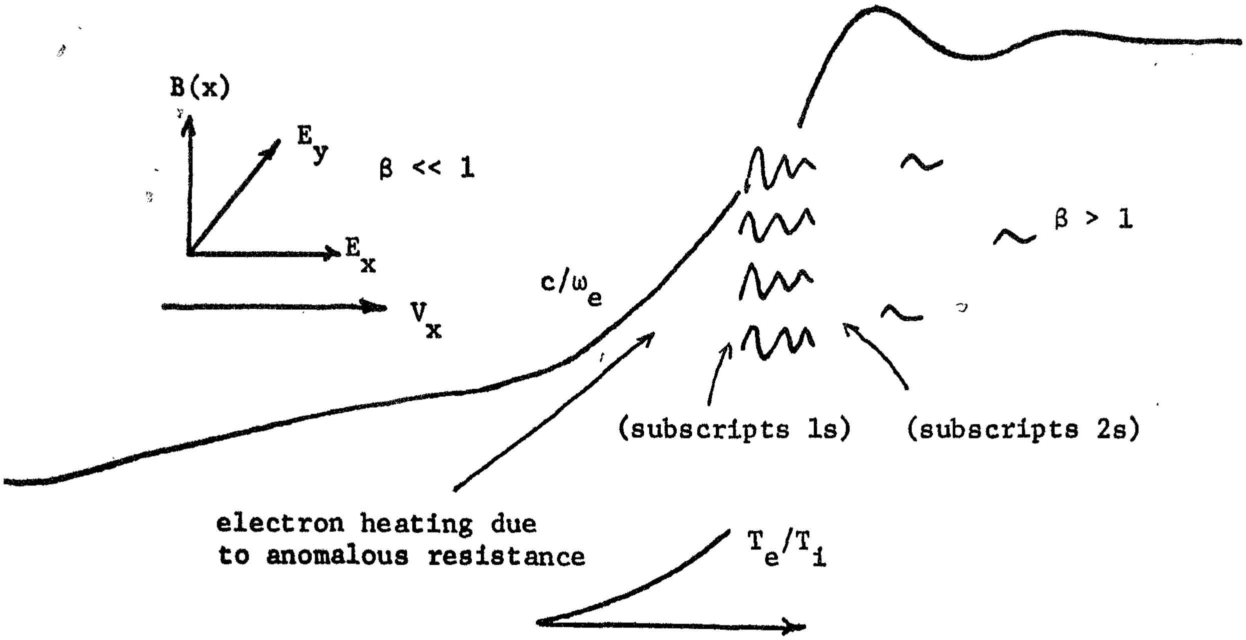
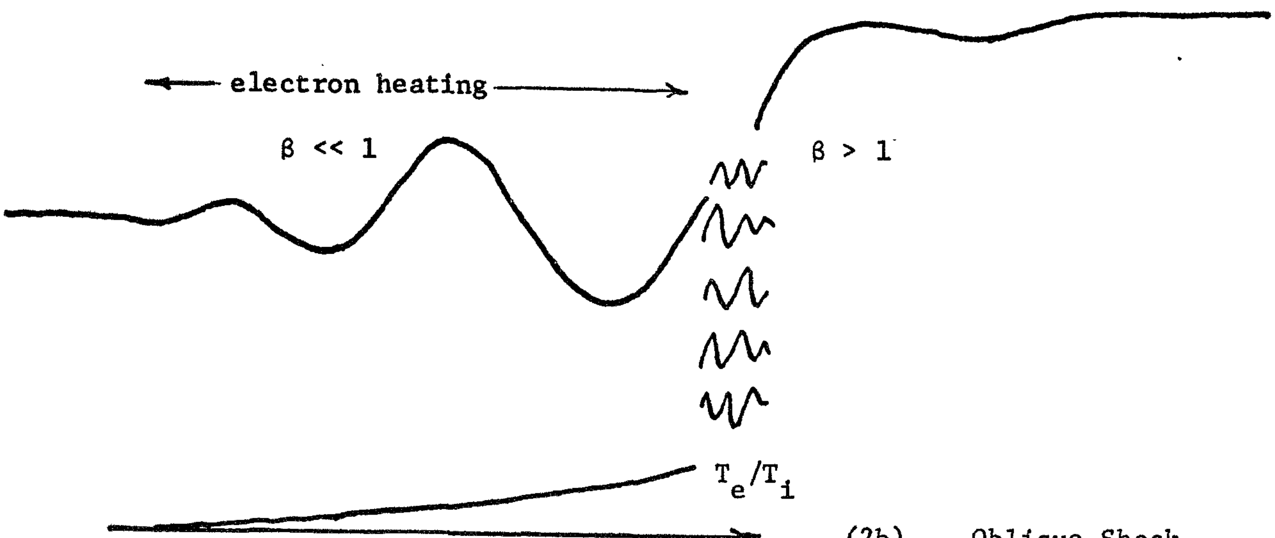


Figure 1.

Schematic plot of the consequences of unstable ion distributions resulting from ion reflection in a magnetosonic shock wave close to the wave-breaking Mach number.



(2a) , Magnetosonic Shock



(2b) , Oblique Shock

Figure 2.

Electrostatic Shock transitions in magnetosonic and whistler shocks. The separation of length scales between the magnetic and electrostatic transitions is similar to the high-beta case discussed in reference[1] .